PREDICTION OF GAS LEAKAGE THROUGH CLEARANCES IN SCROLL COMPRESSORS

Evandro L.L. PEREIRA, Cesar J. DESCHAMPS
POLO Research Labs for Emerging Technologies in Cooling and Thermophysics,
Federal University of Santa Catarina, Florianópolis, 88040-900, Brazil
deschamps@polo.ufsc.br

ABSTRACT

Gas leakage is an important source of inefficiency in scroll compressors and has to be estimated in simulation models. Current models and correlations are usually based on simplified flow conditions. As a consequence, there remains much uncertainty about such predictions for wide range of operating conditions found in actual applications. This paper reports a numerical analysis of radial and axial leakages in scroll compressors covering different operating conditions and geometric parameters. Correlations for gas leakage are obtained and validated through comparisons with data available in the literature.

1. INTRODUCTION

The scroll compressor is well known for its efficiency in applications of high refrigeration capacities. An advantage inherent of scroll compressors in comparison with other compression mechanisms is the longer time available for the suction and discharge processes. On the other hand, gas leakages through different paths (Fig. 1) are major sources of thermodynamic irreversibility. Radial leakage occurs through the axial clearance between the tip of the involute and the opposite scroll base (Fig. 1a). The other path is formed by the radial clearance between the flanks of the two scrolls, resulting in tangential leakage (Fig. 1b). Leakage leads to reduction of isentropic efficiency since a portion of the energy used to compress the gas is dissipated when the gas leaks from one chamber to the other. Moreover, hotter gas arriving in a chamber via leakage is compressed again, consuming extra amount of energy and contributing to increase the temperature of the gas in that chamber.

Basically, four modeling approaches are reported in the literature to estimate leakage in scroll compressors: i) isentropic flow model; ii) incompressible viscous flow model; iii) adiabatic compressible viscous flow model (Fanno flow model); iv) quasi-one-dimensional model. The model of isentropic flow in convergent nozzle is the simplest approach and accommodates compressibility effects. Viscous and geometric effects are included through the flow contraction coefficient, $C_f$, whose value is usually adjusted with reference to experimental data (Kim et al., 1998; Cho et al., 2000). Considerable errors can be expected in small clearances and when entrance losses are important, such as in the radial leakage (Zuk et al., 1972). Ishii et al. (1996) adopted a model of incompressible viscous flow based on correlations of Darcy-Weisbach to predict leakage. An iterative solution procedure is necessary in their model due to the dependency between the friction factor and mass flow rate, characterized by the Reynolds number. The authors adjusted and validated the model for some gases and operating conditions.

In the Fanno flow model, the viscous effects are estimated via the Moody friction factor. As the Mach number of the flow is not known, an iterative solution procedure is necessary. An analytical solution is possible for the radial leakage, but the tangential leakage requires a numerical solution because the dimension of the gap varies along the flow (Suefuji and Shiibayashi, 1992; Fan and Chen, 1994). An adopted approach is to consider the gap to be constant and adjust its length to obtain the same viscous friction loss (Yanagisawa and Shimizu, 1985). The quasi-one-dimensional model adopts the simplifying assumptions of boundary layers and incorporate the changes in flow area, providing a relationship between the pressure drop and volumetric flow rate along the clearance (Yuan et al., 1992; Huang, 1994). This class of model incorporates viscous and inertial effects, requiring a numerical solution procedure.
Prins and Infante-Ferreira (1998a) presented a comparative analysis of leakage predictions given by four models based on the quasi-one-dimensional formulation: (i) isentropic flow model; (ii) model of Ishii et al. (1996) for incompressible viscous flow; (iii) model of Yuan et al. (1992) for compressible viscous flow and (iv) model of Anderson (1995) for compressible viscous flow. The authors used the experimental data of Ishii et al. (1996) and Pevelin (1988) to assess the performance of such models and found none of them was applicable in all conditions. In other study, Prins and Infante-Ferreira (1998b) numerically estimated leakage by using the Fanno flow model and data of Peveling (1988) to calibrate an expression for the friction factor. However, only a slight improvement was found in relation to leakage predictions given by the model of Ishii et al. (1996).

As can be seen from the literature review, there is no clear evidence about the most appropriate model to predict leakage in scroll compressors. The isentropic flow model seems to be the most widely adopted, with contraction coefficients defined via calibration to experimental data. Incompressible and compressible viscous models are also used and require calibration of friction factors. This paper reports a numerical analysis of radial and tangential leakages in scroll compressors and the development of correlations suitable for lumped models, considering different operating conditions and geometric parameters.

2. MATHEMATICAL MODEL AND SOLUTION PROCEDURE

The governing equations (mass, momentum and energy) of the compressible, turbulent flow were solved for averaged quantities. The SST turbulence model was selected to estimate the turbulent transport following the eddy viscosity concept, $\mu_t$, given its accuracy for rapidly strained flows. The simulation model was developed with a commercial code based on the finite volume method (ANSYS, 2010). A second-order upwind scheme was adopted to interpolate the flow quantities required at the faces of each cell of the computational grid. The coupling between the pressure and velocity fields was achieved with the PISO scheme.

The viscous wall region was modeled with the enhanced wall treatment combined with a two-layer model, in which the cells are split into a viscosity affected region and a fully turbulent region. Following the two-layer approach, the SST turbulence model is employed in the fully turbulent region ($y^+ > 30$). In the viscosity affected near wall region the one-equation model of Wolfstein (1969) is adopted instead.

The grid refinement in the viscous wall region ($y^+ < 30$) is essential to resolve flow property gradients. In order to properly solve this region, two criteria were established: i) the values of $y^+$ for grid cells adjacent to the solid walls should be close to 1; ii) the viscous wall region should be discretized with at least 10 grid cells. Moreover, the growth of the cell size near the wall region was limited to 10%. The Grid Convergence Index (GCI) derived from the theory of generalized Richardson Extrapolation (Roache, 1998) was adopted to verify grid convergence, i.e., that further grid refinement did not produce a variation greater than 3% in the mass flow rate.

2.1. Solution Procedure for Radial Leakage

The length of the involute curve that forms the compressor chambers is typically much larger than its thickness, except near the central region of the scroll. This allows the curvature to be neglected and the
analysis can be simplified to a two-dimensional flow. Figure 2a illustrates the solution domain adopted in the two-dimensional modeling of the radial leakage through the tip clearance. Unlike the work of Huang (1994), the opposite sidewalls of the chamber are located at an equal distance \( r_0 \) from the clearance, which corresponds to the radius of the circular orbit. The computational domain represents half of the chamber height. The boundary conditions of pressure, \( p_n \), and temperature, \( T_h \), are average values in the chamber and are imposed on the mean scroll height \((h/2)\). Preliminary analyses showed little sensitivity of leakage to typical values of \( r_0 \) and \( h \). Thus, average values were adopted \((t/h = 1.5, h/\alpha = 10; \text{where } \alpha \text{ is the radius of base circle})\). The dimensionless clearance, \( \delta^* = \delta_c/t \), was the parameter varied in the analysis.

In addition to the boundary conditions of pressure and temperature \((p_n, T_h)\), turbulence intensity \((I = 3\%)\) and length scale \((L = 0.07 r_0)\) were also prescribed at the inlet. Identical conditions for turbulence were imposed at the outlet boundary for the case of reversal flow. The discretization of the solution domain allowed the solution of the viscous layer near the wall. The solution procedure was assumed converged when the relative change of mass flow rate over the past fifty iterations was less than \(10^{-5}\).

2.2. Solution Procedure for Tangential Leakage

Unlike other models in the literature that simplify the flank geometry by two circular curves of constant radius, this study considered the actual variation of the curvature. The code developed to generate the computational mesh restricted the solution domain to two adjacent half-chambers, as illustrated in Figure 2(b). Thus, the dimensionless clearance \((\delta^*_f = \delta_f/D_h)\) and curvature \((C^* = D_h/R_c)\) parameters define the flow geometry to be analyzed. The parameter \(R_c\) is the average radius of the most central chamber, i.e., the high-pressure chamber. The hydraulic diameter \(D_h\) for a two-dimensional flow \((h \gg \alpha)\) is equivalent to \(D_h = 2r_0\). All other considerations made for the radial leakage model (boundary conditions, mesh refinement, convergence criteria, etc.) were also adopted in the modeling of tangential leakage.

3. RESULTS

Dimensional analysis was adopted to reduce the number of variables in the problem, thus reducing the number of simulations to a minimum. In addition to the dimensionless groups required to ensure geometric similarity, the following dimensionless groups were defined for dynamic similarity:

\[
M = \frac{m'}{\delta_p \sqrt{\gamma RT_h}} \quad \Pi = \frac{p_h}{p_n} \quad P = \frac{p_h \delta^2}{10^7 \mu^2}
\]  

In the equations above, \( \delta \) is either the tip clearance, \( \delta_c \), or flank clearance, \( \delta_f \), and \( R \) is the gas constant. A factor equal to \(10^7\) is used to limit the maximum value for \( P \). The parameter \( M \) is the characteristic Mach number at the clearance entrance and it is related to the mass flow rate per unit length. The parameter \( \Pi \) is...
the ratio of the downstream pressure, \( p_l \), and upstream pressure, \( p_h \). Finally, \( P \) can be understood as a potential for leakage, associated with forces due to pressure and viscous friction. The ratio of specific heats \( \gamma \) is also required in the compressible flow analysis. Physically, \( \Pi \) varies between 0 and 1. For typical conditions of interest, \( P \) is within the range from \( 10^{-3} \) to \( 10^{3} \), although values between 1 and 100 are most commonly observed. For refrigerants, \( \gamma \) assumes values between 1.10 and 1.40, with values around 1.20 for most gases.

The asymptotic increase of \( M \) as the pressure ratio \( \Pi \) is decreased is well represented by the Richards growth function (Richards, 1959):

\[
M = \left( \frac{\hat{f}_1}{q \left( 1 + q \exp\left[ -\hat{f}_2 \left( 1 - \Pi \right) \right] \right)} - 1 \right) \times f_\gamma
\]  

(2)

where \( q \) (= 0.01) is a form parameter that changes the shape of the curve. Function \( f_\gamma \) (=0.4 \( \gamma \) + 0.56) corrects the value of \( M \) according to the ratio of specific heats \( \gamma \). On the other hand, \( f_1 \) and \( f_2 \) are functions of dimensionless parameters that depend on the clearance under analysis. In the case of the radial leakage,

\[
f_1(P, \delta^*_t) = \frac{0.53}{1 + g_1\left( \delta^*_t \right)^{-0.80}} \]  

(3)

\[
f_2(P, \delta^*_t) = \frac{10}{1 + g_2\left( \delta^*_t \right)^{-0.56}} \]  

(4)

where functions \( g_1 \) and \( g_2 \) are obtained from

\[
g_i(P) = c_{1,i} + \frac{c_{2,i} - c_{1,i}}{1 + qP^{\delta^*_t}} \]  

(5)

The parameters \( c_{1,i}, c_{2,i}, \) and \( c_{3,i} \) are given in Table 1, according to two different ranges of values for the dimensionless parameter \( P \).

Functions \( f_1 \) and \( f_2 \) assume different forms for tangential leakage:

\[
f_1(C^*, P, \delta^*_t) = \frac{0.59}{1 + g_1\left( \delta^*_t \right)^{-0.469}} \]  

(6)

\[
f_2(C^*, P, \delta^*_t) = g_2\left( \delta^*_t \right)^{-0.413} + 1.9 \]  

(7)

with \( g_1 \) and \( g_2 \) being expressed by

\[
g_1(C^*, P) = h_{1,1}P^{h_{1,2}} + h_{1,3} \]  

(8)

\[
g_2(C^*, P) = \frac{h_{2,1}}{1 + h_{2,2}P^{h_{2,3}}} \]  

(9)
The parameters \( h_{1,1}, h_{1,2}, h_{1,3}, h_{2,1}, h_{2,2} \) and \( h_{2,3} \) in eqs. (8) and (9) are written in the general form

\[
h_{ij}(\gamma^*) = c_{1,ij} \ln(\gamma^* + c_{2,ij}) + c_{3,ij}
\]

(10)

The parameters \( c_{1,ij}, c_{2,ij} \) and \( c_{3,ij} \) are given in Table 2, also for two intervals of values for \( P \). The constants in Tables 1 and 2 were obtained with the Levenberg–Marquardt curve-fitting algorithm.

The proposed correlations for radial and tangential leakages were validated through comparisons with experimental data. Given the few measurements specifically carried out in clearances of geometries similar to those found in scroll compressors, experimental data of general studies of gas flow through microchannels were also used.

<table>
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<th>( i )</th>
<th>( c_{1,i} )</th>
<th>( c_{2,i} )</th>
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Table 2. Parameters \( c_{1,ij}, c_{2,ij} \) and \( c_{3,ij} \) in eq. (10) for tangential leakage.

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<th>( i )</th>
<th>( j )</th>
<th>( c_{1,ij} )</th>
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3.1. Prediction of Radial Leakage

Zuk et al. (1972) presented numerical and experimental results for air flow through microchannels (height \( \delta = 38-39 \) \( \mu \)m, length \( l = 6.4 \) mm, width \( L = 9.55 \) mm) under different pressure ratios \( \Pi \). The measurements were carried out in a radial diffuser geometry and the numerical modeling considered the simplified geometry of flow between parallel plates, which is similar to the geometry adopted in the present study. The numerical results of Zuk et al. (1972) were obtained by solving the Fanno flow equations, with and without the presence of minor loss at the channel entrance.
Figure 3a shows that predictions of radial leakage given by eq. (2) are in good agreement with the experimental and numerical results of Zuk et al. (1972) at three test conditions. It is also evident that the correlation is capable of predicting the radial leakage even when the pressure drop at the entrance is considerable (Test 3). It is also noteworthy that the numerical results obtained by Zuk et al. (1972) agree with measurements when a contraction loss coefficient, $C_d$, equal to 0.6 is used to correct the stagnation properties at the channel entrance.

Suefuji et al. (1992) measured the mass flow rate of R22 through typical clearances of scroll compressors (height $\delta = 3.6-21.0 \, \mu m$, length $t = 4.5 \, mm$, width $L = 126 \, mm$). The inlet pressure was varied during the tests, as well as the clearance dimension, but the authors did not inform the gas temperature at the entrance. In the present study, the gas at the gap entrance was assumed to be at ambient temperature ($T_h = 300 \, K$). For the three clearances analyzed, satisfactory agreement was observed between predictions and measurements, as indicated in Figure 3b.

Shi et al. (2001) studied the flow of air in clearances formed by parallel plates (height $\delta = 1.0 \, mm$, length $t = 120 \, mm$, width $L = 50.5 \, mm$). Experimental data were obtained for geometries with dimensions larger than those found in scroll compressors, but with the ratio $\delta/t$ similar to that of the tip clearance. Figure 4 shows that numerical results provided by formulations of turbulent flow (SST model) and laminar flow are in good agreement with the data of Shi et al. (2001). In fact, it is worthwhile to note that the SST model is also capable of predicting laminar flow regime when the Reynolds number is sufficiently low. The greatest difference between the results of both formulations occurs in the test condition A, in which the flow is probably in the laminar-turbulent transition regime, with a Reynolds number of approximately 7000.

3.2. Prediction of Tangential Leakage

Yuan et al. (1992) studied numerically and experimentally the flow of R-12 and R-22 through a simplified geometry of radial clearance (height $\delta = 4-13 \, \mu m$, curvature $C* = 0.5$, height $h = 21.4 \, mm$). Subsequently, Fan and Chen (1994) used the same apparatus to include $N_2$ in the analysis.

As shown in Figure 5, the agreement between the measurements of those authors and the present correlation is satisfactory, except when the upstream pressure is increased in the case of $N_2$ (Figure 5a). It is not an easy task to explain the reason of this discrepancy. For instance, wall velocity is not included in the correlation, but numerical analysis showed its minor effect on leakage. On the other hand, assembly misalignments, manufacturing tolerances and deformation due to pressure load are important sources of measurement uncertainty, which was not reported by Fan and Chen (1994).
Leakage is a major source of thermodynamic inefficiency of scroll compressors and many theoretical studies are available in the literature. On the other hand, most models proposed to predict leakage are based on calibration and lack enough experimental data for their validation. The present study reported a numerical analysis of radial and tangential leakages in scroll compressors for different operating conditions, considering the effects of entrance loss on radial leakage and scroll curvature on tangential leakage. New correlations for radial and tangential leakages including both effects were proposed and validated through comparisons with measurements available in the literature. Naturally, other aspects can be included to complement the present analysis, such as the presence of lubricating oil and wall surface finishing.

5. ACKNOWLEDGEMENTS

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